# Solution to Ex. 7.15

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Note that it is assumed the following relation holds on the axis.

$$u^+ \equiv \frac{\langle U \rangle}{u_\tau} = \frac{1}{\kappa} \ln\left(\frac{y u_\tau}{\nu}\right) + B \tag{1}$$

and we know that the mean velocity profile can be approximated by the logarithmic defect law (Eq. (7.104) in the book)

$$\frac{U_0 - \langle U \rangle}{u_\tau} = -\frac{1}{\kappa} \ln\left(\frac{y}{R}\right) \tag{2}$$

and yet

$$\frac{U_0 - \bar{U}}{u_\tau} = \frac{3}{2\kappa} \tag{3}$$

follow the analysis procedures of the channel flow, if we add up Eq. (1) and Eq. (2), we get

$$\frac{U_0}{u_\tau} = \frac{1}{\kappa} \ln\left(\frac{y}{\delta_\nu}\right) + B \tag{4}$$

here $y$ has the same meaning of $\delta$, so

$$\frac{y}{\delta_\nu} = \frac{\delta}{\delta_\nu} = \mathrm{Re}_\tau = \frac{u_\tau \delta}{\nu} \tag{5}$$

make use of Eq. (7.104) on the book, we get

$$\mathrm{Re}_\tau = \frac{2\bar{U}\delta}{\nu} \frac{\sqrt{f}}{4\sqrt{2}} = \mathrm{Re}\frac{\sqrt{f}}{4\sqrt{2}} \tag{6}$$

from Eq. (7.107) on the book, the relationship between $U_0$ and $u_\tau$ in Eq. (7.107) could be written

$$U_0 = \frac{3}{2\kappa} u_\tau + \bar{U} \tag{7}$$

by using Eq. (7) and Eq. (6), Eq. (4) can be written as

$$\frac{3}{2\kappa} + \frac{\bar{U}}{u_\tau} = \frac{1}{\kappa} \ln\left(\mathrm{Re}\frac{\sqrt{f}}{4\sqrt{2}}\right) + B \tag{8}$$

use Eq. (7.104) again, Eq. (8) can be changed into

$$\frac{1}{\sqrt{f}} = \frac{1}{2\sqrt{2}\kappa} \ln\left(\mathrm{Re}\sqrt{f}\right) - \frac{5\ln 2 - 2\kappa B + 3}{4\sqrt{2}\kappa} \tag{9}$$