Solution to Ex. 6.30

of *Turbulent Flows* by Stephen B. Pope, 2000

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Show that in isotropic turbulence, the longitudinal integral scale is

$$L_{11} = \frac{\pi}{2\langle u_1^2 \rangle} \int_0^\infty \frac{E(\kappa)}{\kappa} \, \mathrm{d}\kappa \tag{1}$$

Solution

From Eq. (6.213) and Eq. (6.216)

$$\begin{aligned} L_{11} &= \frac{\pi E_{11}(0)}{2\langle u_1^2 \rangle} \\ &= \frac{\pi}{2\langle u_1^2 \rangle} \int_0^\infty \frac{E(\kappa)}{\kappa}\left(1 - \frac{0}{\kappa^2}\right) \mathrm{d}\kappa \\ &= \frac{\pi}{2\langle u_1^2 \rangle} \int_0^\infty \frac{E(\kappa)}{\kappa} \, \mathrm{d}\kappa \end{aligned} \tag{2}$$