

Solution to Ex. 6.22

of *Turbulent Flows* by Stephen B. Pope, 2000

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Show that $\hat{E}(\boldsymbol{\kappa})$ (Eq. (6.158)) is real, non-negative, with

$$\hat{E}(\boldsymbol{\kappa}) = \hat{E}(-\boldsymbol{\kappa}) \quad (1)$$

Solution

From definition of $\hat{E}(\boldsymbol{\kappa})$ we can easily write that

$$\hat{E}(\boldsymbol{\kappa}) = \frac{1}{2} \langle \hat{u}_i^*(\boldsymbol{\kappa}, t) \hat{u}_i(\boldsymbol{\kappa}, t) \rangle = \frac{1}{2} \langle \hat{u}_i(-\boldsymbol{\kappa}, t) \hat{u}_i^*(-\boldsymbol{\kappa}, t) \rangle = \hat{E}(-\boldsymbol{\kappa}) \quad (2)$$