Solution to Ex. 6.14

of *Turbulent Flows* by Stephen B. Pope, 2000

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Given that the volume-average velocity $\langle \mathbf{u}(\mathbf{x},t) \rangle_L$ is zero, show that the coefficient of the zeroth Fourier mode is zero:

$$\hat{\mathbf{u}}(0,t) = 0 \tag{1}$$

Solution

The zeroth Fourier mode of a velocity component is expressed as

$$\begin{aligned} \hat{u}_j(0,t) &= F_0\{u_j(\mathbf{x},t)\} \\ &= \langle u_j(\mathbf{x},t) e^{-i\{0\}\cdot\mathbf{x}} \rangle_L \\ &= \langle u_j(\mathbf{x},t) \rangle_L \\ &= 0 \end{aligned} \tag{2}$$

where $\{0\} = \boldsymbol{\kappa}$ is a zero wavenumber vector vector. From Eq. (2), it is obvious that Eq. (1) holds.