Solution to Ex. 13.12

of *Turbulent Flows* by Stephen B. Pope, 2000

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Consider the LES of high-Reynolds-number isotropic turbulence using a pseudo-spectral method and the sharp spectral filter with cutoff wavenumber $\kappa_c = \kappa_r$ in the inertial subrange. Show that, if 90% of the energy is to be resolved, then a factor of

$$2^{\frac{9}{2}} = 23 \tag{1}$$

more nodes is required than would be needed if only 80% of the energy were to be resolved.

Solution

Based on the discussion of Ex.13.10 and Ex.13.11, for sharp spectral filter, an estimate for the fraction of the energy in the residual motions could be expressed as

$$\frac{\langle k_r \rangle}{k} = \frac{3}{2} C \left(\kappa_c L \right)^{-\frac{2}{3}} \tag{2}$$

If 80% energy is resolved then

$$\frac{\langle k_r \rangle}{k} = 0.2 \tag{3}$$

Now we want 90% energy to be resolved, and it means that Eq. (3) equals 0.1. Then the new cutoff wavenumber, $\kappa_{c,90}$, should give

$$\frac{3}{2} C \left(\kappa_{c,90} L \right)^{-\frac{2}{3}} = \frac{1}{2} \frac{3}{2} C \left(\kappa_c L \right)^{-\frac{2}{3}} \tag{4}$$

It clear that

$$\kappa_{c,90} = 2^{\frac{3}{2}} \kappa_c \tag{5}$$

Recall that the cubic of κ_c and and the grid number N are proportional if $\kappa_c = \kappa_r$. From Eq. (5) we know that if 90% energy is to be resolved, then the grid number should be

$$\left(2^{\frac{3}{2}}\right)^3 = 2^{\frac{9}{2}} = 22.6 \qquad (6)$$

times the original grid number.